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A STUDY OF THE PHASE TRANSFORMATION AND THE CHARACTERISTIC LINES OF SAND BEHAVIOR

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ABSTRACT

Experimental data from several types of sand have been analyzed to study and compare the phase transformation line and the characteristic line of sand behavior. The definitions of the two lines are reviewed, and the factors that may affect the two lines have been investigated. These include the relative density, the minor and the intermediate principal stresses. It is shown that the relative density does not influence either of the two lines, the lines go through the stress origin, and both are curved. Most data indicate that the two lines are identical, and that the minor and the intermediate principal stresses affect the slopes of the lines.

KEYWORDS: characteristic state, confining pressure, cubical triaxial test, dilatancy, drained behavior, phase transformation state, sand, shear test, triaxial test, undrained behavior.

INTRODUCTION

Volume changes are important for the behavior of sands whether under drained or undrained conditions and under static and cyclic loading. Volume changes can be compressive or expansive in nature. Expansive or dilative volume changes are most pronounced for dense sands at low confining pressures and high stress levels approaching failure. The transition from compressive behavior observed at lower stress levels to dilative behavior at high stress levels occurs along a line through the origin of the stress space. This line is referred to as the phase transformation line (for undrained tests) or the characteristic line (for drained tests).

In development of constitutive models, it is important to capture the location of the phase transformation line correctly in order that the transition from compressive to dilative behavior be modeled correctly. In elasto-plasticity models this is done by the plastic potential function. Thus, the phase transformation line plays a similar role for the plastic potential surface as the instability line plays for the yield surface. The soil behavior is greatly influenced by and may be explained in view of the locations of these two lines.

Experimental evidence is reviewed to study whether the phase transformation line and the characteristic line are identical and to investigate the factors that potentially could influence their shape and location for various types of sands. These factors include the relative density, the minor principal stress, and the intermediate principal stress.

SHEAR STRENGTH OF SAND

The typical variation of the drained shear strength of sand with confining pressure is illustrated schematically in Fig. 1. For a sand with a given initial density the shear strength is composed of two contributions: (1) one from the basic friction between sand particles modified by contributions for rearrangement of particles at constant volume. The resulting strength is calculated from the critical friction angle. (2) The second contribution derives from the dilation of the sand during shear. Due to crushing the dilation is suppressed at higher pressures and the contribution due to the rate of dilation reduces to zero at very high pressures. Thus, a curved failure surface is observed.

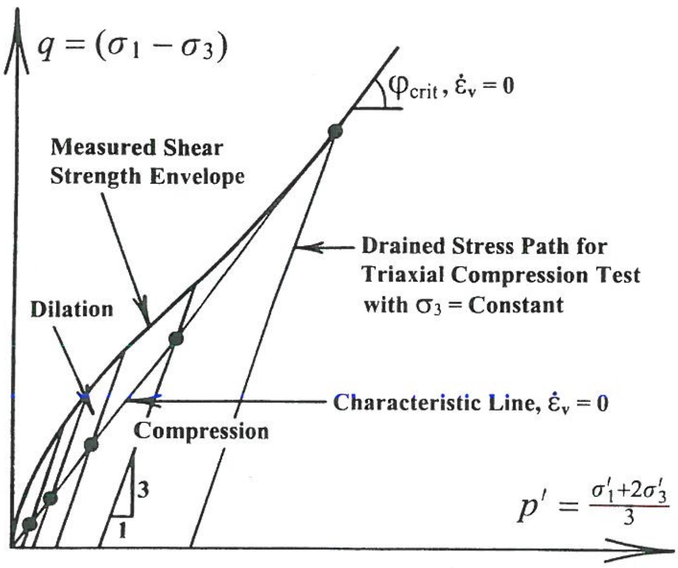

Fig. 1. Variation of drained shear strength envelope for sand with confining pressure.

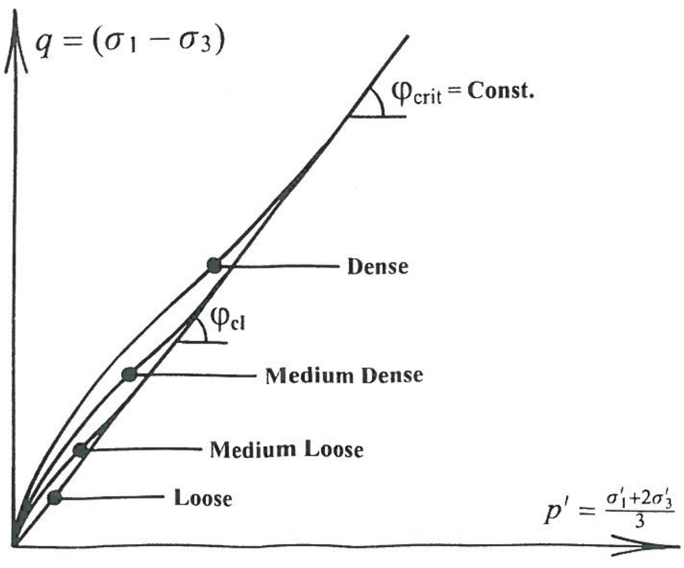

Fig. 2. Variation of drained shear strength envelope for sand with relative density.

Experiments on sands have shown that (1) the contribution from dilation reduces, and (2) the range of confining pressures in which dilation occurs reduces with decreasing relative density, as shown schematically in Fig. 2.

THE CHARACTERISTIC LINE

The separation between the region of compression and the region of dilation for drained tests on sand occurs at the characteristic state at which the rate of volume change is zero, $\dot{\varepsilon}_v=0$ (Luong 1982), as shown schematically in Fig. 3. Characteristic states occur at the transition from compression to dilation, and these states are located on a line through the stress origin, whose slope may be characterized by an angle, φ_{cl}. The characteristic state and the critical state are very similar, as discussed by Luong (1982). For loose sand and sand at high confining pressure, the $\dot{\varepsilon}_v=0$ state is reached at the critical state. The critical state is therefore the same as the characteristic state, and it occurs at failure for sand that compresses during shear. For dense sand or sand at low confining pressure, the characteristic state is reached at small strain magnitudes, as indicated in Fig. 3, while the critical state is reached at large strains.

Effects of Relative Density and Minor Principal Stress

The stress states corresponding to $\varepsilon_v=0$ for triaxial compression tests on Santa Monica Beach sand at four relative densities (Lade and Prabucki 1995) are shown in Fig. 4. These experiments were performed on specimens with height-to diameter ratio H/D=2.65 and with lubricated ends. They show that the characteristic angle φ_{cl} is independent

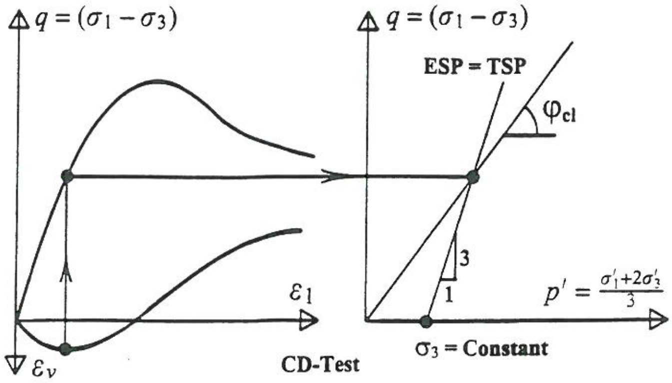

Fig. 3. Schematic diagram of characteristic state in drained triaxial compression test on sand.

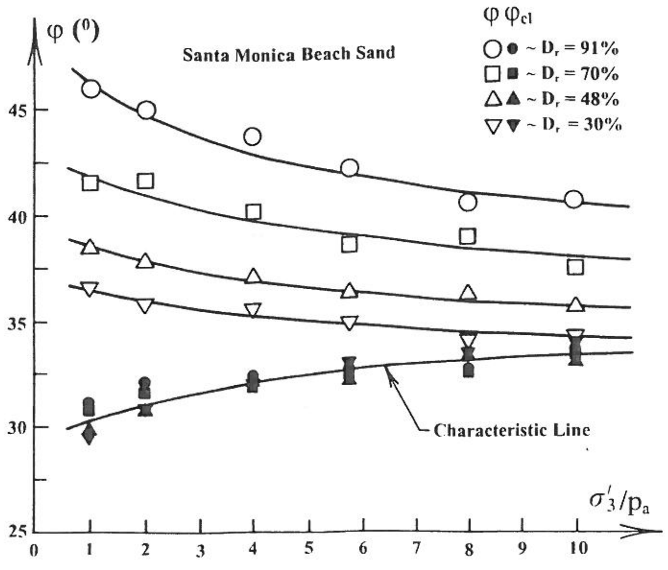

Fig. 4. Characteristic states obtained from triaxial compression tests on Santa Monica Beach sand.

of relative density for a given sand, and that it to varies with confining pressure or minor principal stress.

The characteristic angles for drained triaxial compression tests on Sacramento River sand (Lee 1965, Lee and Seed 1967) were also determined for four different relative densities, and they are shown in Fig. 5. These tests were performed on specimens with D=3.56 cm and H/D=2.43, and without lubricated ends. Determination of the stress state at $\dot{\varepsilon}_v=0$ is not necessarily very accurate, because the volume change curve is relatively flat near the characteristic state, while the stress state varies considerably. The data for Sacramento River sand

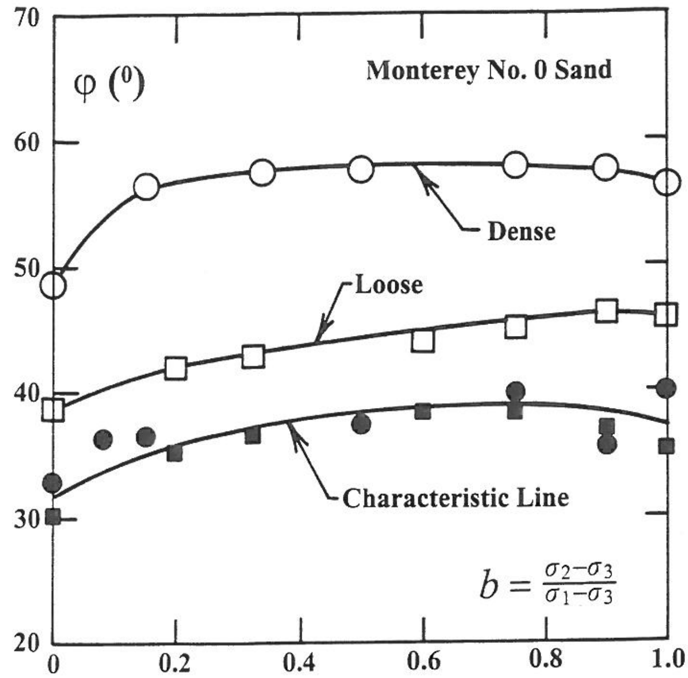

Fig. 6. Characteristic states obtained from cubical triaxial tests on Monterey No. 0 sand.

therefore shows more scatter, but it indicates that the characteristic angle is independent of relative density, but it varies more with confining pressure than the data for Santa Monica Beach sand.

Effects of Intermediate Principal Stress

The characteristic angles were deduced from drained cubical triaxial compression tests on dense and loose Monterey No.0 sand (Lade and Duncan 1973) and shown in Fig. 6. The characteristic angles are affected by the intermediate principal stress in a similar fashion but to a lesser degree as

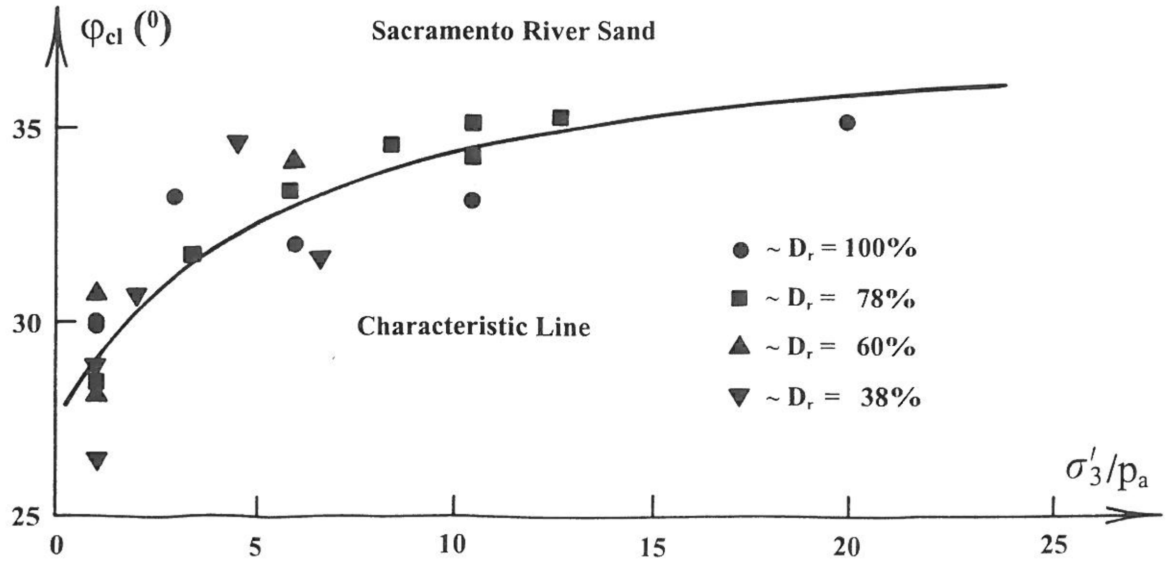

Fig. 5. Characteristic states obtained from triaxial compression tests on Sacramento River sand.

the measured friction angles. However, there does not seem to be a pronounced effect of relative density on the characteristic angles.

THE PHASE TRANSFORMATION LINE

The phase transformation line plays a similar role for undrained tests as the characteristic line plays for drained tests. Fig. 7 shows a schematic illustration of the stress state at which phase transformation occurs along an effective stress path from an undrained test. It is the point at which "the stress path turns its direction in p'-q space" (Ishihara et al. 1975), i.e. the point where the effective stress path has a "knee" and the effective mean normal stress reaches a minimum value. Ishihara et al. (1975) observed that for cyclic undrained triaxial tests "it is necessary for a sample to go at least once through this critical value in order to be taken to a completely liquefied state." In this sense, "the critical stress ratio may be considered as a threshold at which the behavior of sand as a solid is lost and transformed into that of a liquefied state." Note that the stress state corresponding to maximum pore water pressure occurs slightly later than the phase transformation, as also indicated in Fig. 7.

Importance of Phase Transformation Line and Instability Line

The effective stress path has a vertical tangent at the phase transformation point, and it corresponds to the transition from compressive to dilative behavior. At this point, the increment in p' becomes zero and the elastic volumetric strain

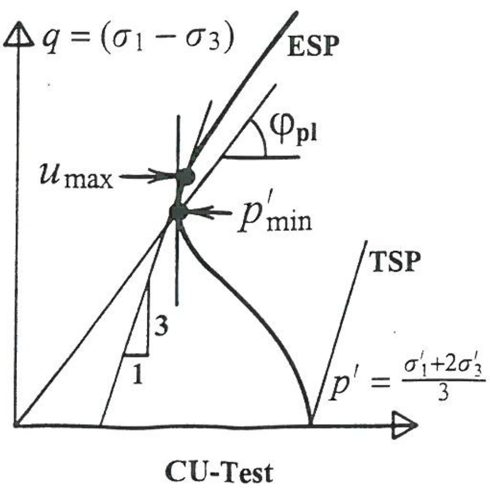

Fig. 7. Schematic diagram of phase transformation state in undrained triaxial compression test on sand.

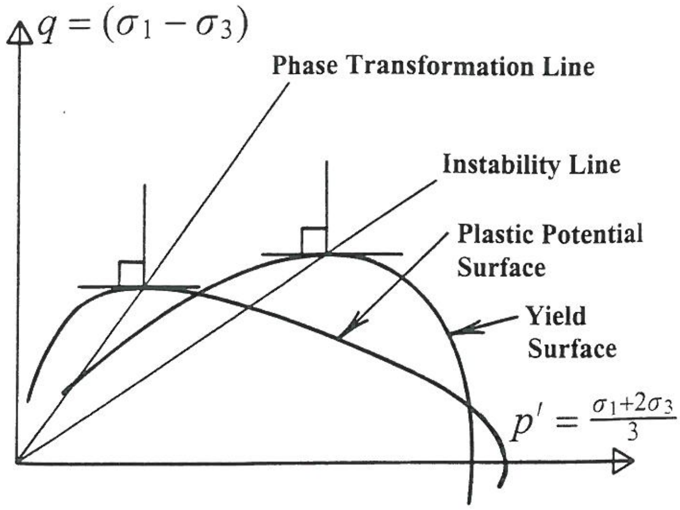

Fig. 8. Comparison of phase transformation and instability lines.

increment is therefore also zero. Consequently, the plastic volumetric strain increment is also zero. In elasto-plasticity models the phase transformation state corresponds to the point on the plastic potential surface where the plastic strain increment vector is perpendicular to the p'-axis or the hydrostatic axis. This state is therefore comparable to the similar point on the yield surface at which the normal is perpendicular to the hydrostatic axis. This indicates the point at which sand may become unstable, as explained in detail elsewhere (Lade 1995). Thus, the phase transformation line plays a similarly important role for the plastic potential surface as the instability line plays for the yield surface. Both lines are shown on the diagram in Fig. 8. The behavior of soils is greatly influenced by and may be explained in view of the relative locations of these two lines. For sands the two lines are distinctly separate, while for normally consolidated, insensitive clays the two lines coincide, and they also coincide with the critical state or ultimate state line. Normally consolidated, insensitive clays do not become unstable and liquefy as sands may do. Rather they reach failure at these lines, and conventional shear failure analyses correctly captures the behavior of these soils.

Effects of Relative Density and Minor Principal Stress

The phase transformation angles, φ_{pl}, are shown in Fig. 9 for undrained triaxial compression tests on Sacramento River sand (Lee 1965, Seed and Lee 1967) for four relative densities. These test were performed under the same conditions as the drained

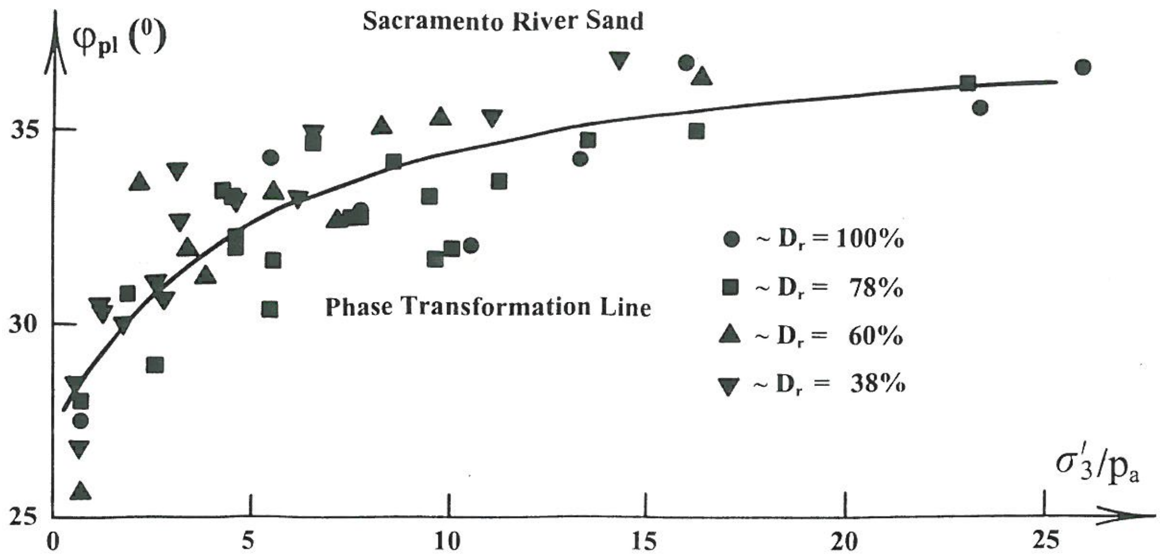

Fig. 9. Phase transformation states obtained from undrained triaxial compression tests on Sacramento River sand.

tests whose characteristic angles are shown in Fig. 5. The solid curved line shown in Figs. 5 and 9 is the same line. It is clear that it represent the variation of the two angles equally well, and this indicates that the two angles are the same for the Sacramento River sand. This data also shows that there is no influence of relative density on the two angles.

If any difference were present between the two lines, it would be expected that the characteristic states would be slightly higher than the phase transformation states, because some elastic compression occurs in the drained tests performed with constant confining pressure rather than constant mean normal stress. However, such an effect is not visible from the data for Sacramento River sand.

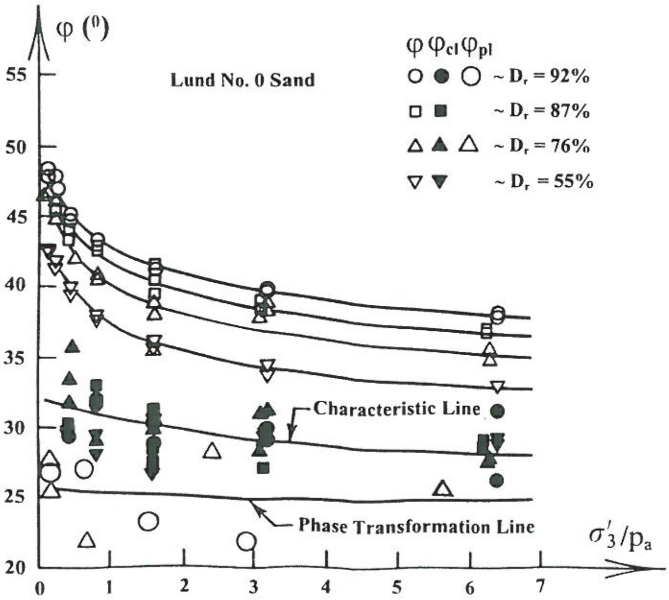

Fig. 10. Comparison of phase transformation and characteristic states obtained from triaxial compression tests on Lund No. 0 sand.

Data from drained and undrained triaxial compression tests on Lund No. 0 sand (Ibsen and Jakobsen 1996) are shown in Fig. 10. These experiments were performed on specimens with H/D=1.0 and with lubricated ends. The friction angles obtained from the drained tests are also shown for comparison on this diagram. The phase transformation and characteristic states exhibit scatter similar to that in Figs. 5 and 9, and the data indicates some difference between the two angles. The characteristic angle is visibly greater than the phase transformation angle, as speculated above. Further, the effect of confining pressure seems to be opposite of that for the other two sands.

Effects of Intermediate Principal Stress

Data from drained and undrained cubical triaxial tests on loose Fuji River sand by Yamada and Ishihara (1979, 1981) were analyzed to determine the characteristic and the phase transformation angles. Fig. 11 show their variations. While some scatter is present, the characteristic angles tend to be higher than the phase transformation angles. The difference is similar in magnitude to that obtained for Lund No. 0 sand, as shown in Fig. 10. The intermediate principal stress influences the two angles in a similar fashion as it influences the measured friction angles. Similar effects were obtained from cubical triaxial tests on Monterey No. 0 sand, as shown in Fig. 6.

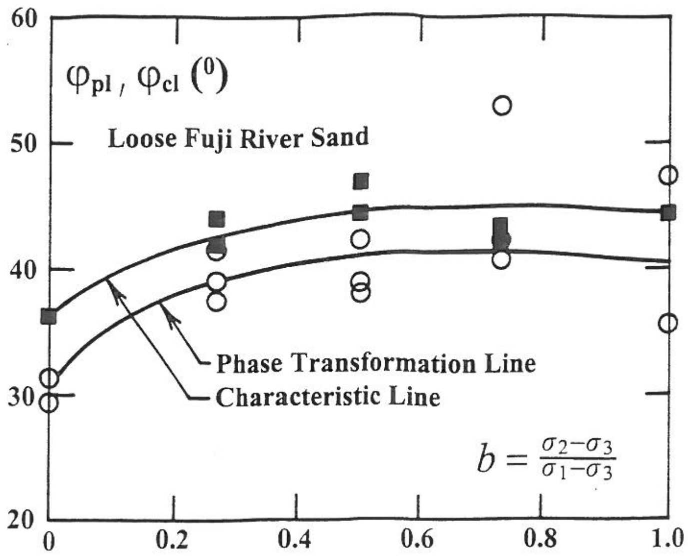

Fig. 11. Comparison of phase transformation and characteristic states obtained from cubical triaxial tests on loose Fuji River sand.

CONCLUSION

The characteristic line and the phase transformation line have been studied in view of experimental results of drained and undrained triaxial compression tests and cubical triaxial tests on different sands. It has been shown that the relative density does not influence the location of the two lines, and these are therefore unique for a given sand. Most data indicate that the two lines are identical, and that the minor as well as the intermediate principal stresses affect the slopes of the lines. Thus, the characteristic line and the phase transformation line are both curved, with slopes increasing with confining pressure. The effect of the intermediate principal stress on the two lines is similar to but less pronounced than its effect on the friction angle.

REFERENCES

Ibsen, L.B. and Jakobsen, F.R. (1996) Lund Sand No. 0, *Data Report 8401, 8402, 8801 & 8901*, Laboratoriet for Fundering, Aalborg Universitetscenter, Aalborg, Denmark.

Ishihara, K., Tatsuoka, F. and Yasuda, S. (1975) Undrained deformation and liquefaction of sand under cyclic stresses. *Soils and Foundations*, **15:1**, 29-44.

Lade, P.V. (1995) Instability of sand in the prefailure hardening regime. *Proc. First Int. Conf. on Pre-Failure Deformation Characteristics of Geomaterials*, **2,** 837-854.

Lade, P.V. and Duncan, J.M. (1973) Cubical triaxial tests on cohesionless soil. *Journal of the Soil Mechanics and Foundations Division*, ASCE, **99:10**, 793-812.

Lade, P.V. and Prabucki, M.-J. (1995) Softening and preshearing effects in sand. *Soils and Foundations*, **35:4**, 93-104.

Lee, K.L. (1965) Triaxial compressive strength of saturated sand under seismic loading conditions, PhD thesis, University of California, Berkeley.

Lee, K.L. and Seed, H.B. (1967) Drained strength characteristics of sands. *Journal of the Soil Mechanics and Foundations Division*, ASCE, **93:6**, 117-141.

Luong, M.P. (1982) Stress-strain aspects of cohesionless soils under cyclic and transient loading. *Int. Symp. on Soils under Cyclic and Transient Loading*, 315-324.

Seed, H.B. and Lee, K.L. (1967) Undrained strength characteristics of cohesionless soils. *Journal of the Soil Mechanics and Foundations Division*, ASCE, **93:6**, 333-360.

Yamada, Y. and Ishihara, K. (1979) Anisotropic deformation characteristics of sand under three dimensional stress conditions. *Soils and Foundations*, **19:2**, 79-94.

Yamada, Y. and Ishihara, K. (1981) Undrained deformation characteristics of loose sand under three-dimensional stress conditions. *Soils and Foundations*, **21:1**, 97-107.

6 Ibsen, L.B. (1995). Soil Parameters, Final proceedings MCS - Project MAST II, July 1995. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9514.

7 Sørensen, C. S., Ibsen, L. B. , Jakobsen, F. R., Hansen, A., Jakobsen, K. P., (1995) "Bearing Capacity of Caisson Breakwaters on Rubble Mounds". Proceedings of the Final Project Workshop, Monolithic (Vertical) Coastal Structures, Alderney, UK, Appendix IX, p 26. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9515.

8 Ibsen, L.B., Steenfelt, J.S. (1996). Terningapparatet - et middel til bedre jordforståelse (The true-triaxial-apparatus - a means to better understanding of soil behaviour; in Danish). *Proc. Nordic Geotechnical Meeting, NGM-96, Reykjavik,* Vol 1, pp 111-122. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9603.

9 Thorsen, G., Thomsen, B., Thorsen, S. (1996). Tilsyneladende forbelastning af Eem jordarter (Apparent preconsolidation of Eemian soils; in Danish). *Proc. Nordic Geotechnical Meeting, NGM-96, Reykjavik,* Vol 1, pp 147-152. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9607.

10 Thorsen, G. (1996). Oedometer tests - an aid in determination of the geological load history. *Bull. of the Geological Society of Denmark*, Vol. 43, pp. 41-50. Copenhagen 1996-07-14. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9608.

11 Ibsen, L.B., Jakobsen, K.P. (1997). Dynamic Bearing Capacity of Caisson Breakwaters Subjected to Impulsive Wave Loading. MAST III (PROVERBS Workshop, Las Palmas, Feb. 18-24-1997. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9701.

12 Lade, P.V., Ibsen, L.B. (1997). A study of the phase transformation and the characteristic lines of sand behaviour. Proc. Int. Symp. on Deformation and Progressive Failure in Geomechanics, Nagoya, Oct. 1997, pp. 353-359. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9702.

13 Bødker, L., Steenfelt, J.S. (1997). Vurdering af lodrette flytningsamplityder for maskinfundament, Color Print, Vadum (Evaluation of displacement amplitudes for printing machine foundation; in Danish). *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9706.

14 Ibsen, L.B., Steenfelt, J.S. (1997). Vurdering af lodrette flytningsamplityder for maskinfundament Løkkensvejens kraftvarmeværk (Evaluation of displacement amplitudes for gas turbine machine foundation; in Danish. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9707.

15 Steenfelt, J.S. (1997). National R&D Report : Denmark. *Seminar on Soil Mechanics and Foundation Engineering R&D*, Delft 13-14 February 1997. pp 4. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9708.

16 Lemonnier, P. and Soubra, A. H. (1997). Validation of the recent development of the displacement method - geogrid reinforced wall. *Colloquy EC97 on the comparison between experimental and numerical results*, Strasbourg, France. Vol.1, pp. 95-102. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9712.

17 Lemonnier, P. & Soubra, A. H. (1997). Recent development of the displacement method for the design of geosynthetically reinforced slopes - Comparative case study. *Colloquy on geosynthetics, Rencontres97, CFG*, Reims, France, Vol. 2, pp. 28AF-31AF (10pp). Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9713.

18 Lemonnier, P., Soubra, A. H. & Kastner, R. (1997). Variational displacement method for geosynthetically reinforced slope stability analysis : I. Local stability. Geotextiles and Geomembranes 16 (1998) pp 1-25. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9714.

19 Lemonnier, P., Soubra, A. H. & Kastner, R. (1997). Variational displacement method for geosynthetically reinforced slope stability analysis : II. Global stability. Geotextiles and Geomembranes 16 (1998) pp 27-44. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9715.

20 Ibsen, L.B. (1998). Analysis of Horizontal Bearing Capacity of Caisson Breakwater. 2nd PROVERS Workshop, Napels, Italy, Feb. 24-27-98. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9802.

21 Ibsen, L.B. (1998). Advanced Numerical Analysis of Caisson Breakwater. *2nd PROVERS Workshop*, Napels, Italy, Feb. 24-27-98. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9803.

22 Ibsen, L.B., Lade P.V. (1998). The Role of the Characteristic Line in Static Soil Behavior. *Proc. 4th International Workshop on Localization and Bifurcation Theory for Soil and Rocks.* Gifu, Japan. Balkema 1998. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9804.

23 Ibsen, L.B., Lade, P.V. (1998). The Strength and Deformation Characteristics of Sand Beneath Vertical Breakwaters Subjected to Wave Loading. 2nd PROVERS Workshop, Napels, Italy, Feb. 24-27-98. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9805.

24 Steenfelt, J.S., Ibsen, L.B. (1998). The geodynamic approach - problem or possibility? Key Note Lecture, *Proc. Nordic Geotechnical Meeting, NGM-96, Reykjavik, V*ol 2, pp 14. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9809.

25 Lemonnier, P., Gotteland, Ph. and Soubra, A. H. (1998). Recent developments of the displacement method. *Proc. 6th Int. Conf. on Geosynthetics. Atlanta, USA*, Vol 2, pp 507-510. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9814.

26 Praastrup, U., Jakobsen, K.P., Ibsen, L.B. (1998). On the choice of strain measures in geo-mechanics. 12th Young Geotechnical Engineers Conference, Tallin, Estonia. *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9815.

27 Ibsen, L.B. (1998). The mechanism controlling static liquefaction and cyclic strength of sand. Proc. Int. Workshop on Physics and Mechanics of Soil Liquefaction, Baltimore. A.A.Balkema, ISBN 9058090388, pp 29-39. Also in *AAU Geotechnical Engineering Papers*, ISSN 1398-6465 R9816.